CAMBRIDGE
International Education

Cambridge O Level

ADDITIONAL MATHEMATICS **4037/21**

Paper 2 **May/June 2026**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme contains the instructions and marking guidance that our examiners used to mark candidate responses for this component.

Before they start marking, examiners complete a standardisation process. They review a range of candidate responses and discuss the mark scheme in detail to agree which answers can and cannot be accepted. Examiners award marks where it is clear that candidate responses have met the requirements of the mark scheme. These requirements may vary between different components or different versions of the same component, depending on the exact requirements of the question and the candidate responses seen during the standardisation process.

Sometimes, our mark schemes may allow marks to be awarded to responses which contain elements that are factually incorrect or for content not covered by the syllabus. We do this in response to the demands of a specific question to support candidates and make sure that our assessments are fair. However, these allowances should not be used as a guide for teaching and learning.

We cannot discuss the mark scheme with schools, and we recommend that schools read the mark scheme together with the assessment material and the Principal Examiner Report for Teachers.

General marking principles

All examiners must apply these marking principles when marking candidate responses.

1	Examiners must award marks for each response in line with: • the specific **content** defined in the mark scheme • the specific **skills** defined in the mark scheme or in the level descriptions • the **standard of response** required as exemplified by the standardisation scripts.
2	Examiners must award **whole marks** (not half marks, or other fractions).
3	Examiners must award marks **positively**. This means that: • marks are not deducted for errors or omissions • marks are awarded for correct/valid answers as defined in the mark scheme and also for **other valid answers** which go beyond the scope of the syllabus and mark scheme referring to your team leader as appropriate • the quality of spelling, punctuation and grammar are judged **only** when the mark scheme indicates that these features are **specifically assessed** in the question. However, the meaning of all responses must be unambiguous.
4	Examiners must **consistently** apply the requirements of the mark scheme and any rules for what to do when candidates have not followed instructions correctly.
5	Examiners must use the **full range of marks** defined in the mark scheme, unless limited by the quality of the candidate responses seen.
6	Examiners must award marks according to the mark scheme and **must not** award marks with grade thresholds or grade descriptions in mind.

Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate’s intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
^	More information required
A0	Accuracy mark awarded zero
A1	Accuracy mark awarded one
A2	Accuracy mark awarded two
A3	Accuracy mark awarded three
B0	Independent mark awarded zero
B1	Independent mark awarded one
B2	Independent mark awarded two
B3	Independent mark awarded three
BOD	Benefit of the doubt
C	Communication mark

Annotation	Meaning
✗	Incorrect
FT	Follow through
Highlighter	Highlight a key point in the working
ISW	Ignore subsequent work
M0	Method mark awarded zero
M1	Method mark awarded one
M2	Method mark awarded two
M3	Method mark awarded three
MR	Misread
O	Omission
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
Pre	Premature rounding/approximation
SC	Special case
SEEN	Indicates that work/page has been seen
TE	Transcription error
✓	Correct

Annotation	Meaning
XP	Correct answer from incorrect working

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

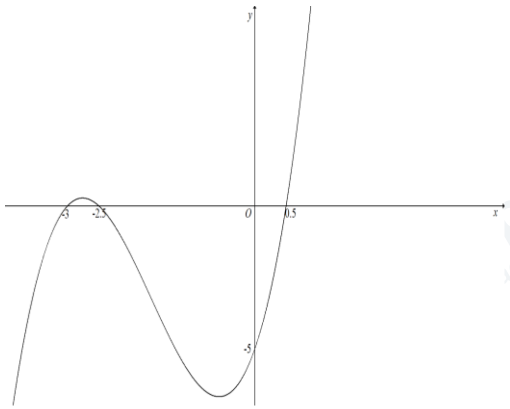
A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

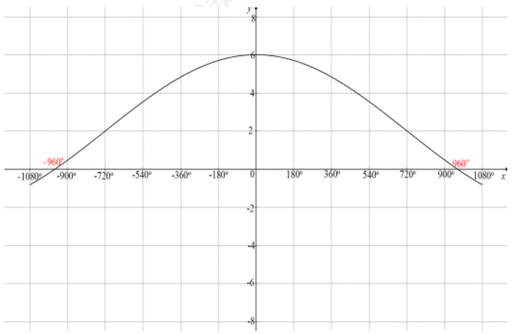
B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation 'dep' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance	Notes
1(a)		3	**B1** for correct shape with max in the 2nd quadrant and min in the 3rd quadrant **B1** for correct x intercepts: –3, –2.5, 0.5; must have attempted the graph of a cubic **B1** for correct y intercept: −5; must have attempted the graph of a cubic	• B0 if minimum is on y-axis • ‘Ends’ of curve must clearly extend beyond x-axis i.e. curve must not stop at $x = -3, x = \frac{1}{2}$ • Accept feathering/ slight doubling back but not heading off towards another turning point • Condone marked to one side of graph, but must be clear they are intercepts e.g. not points marked in a table • Mark better if e.g. intercept at $x = 0.5$ labelled as (0, 0.5) and ‘0.5’ on axis
1(b)	$-3 < x < -\frac{5}{2}$	**B1**		• allow e.g. $-\frac{5}{2} > x > -3$
	$x > \frac{1}{2}$	**B1**	If 0 scored, **SC1** for $-3 \leqslant x \leqslant -\frac{5}{2}$ **and** $x \geqslant \frac{1}{2}$	• For B1B1, condone any conjunction or comma between two inequalities e.g. $-3 < x < -\frac{5}{2}$ ‘and’ $x > \frac{1}{2}$
2(a)	4	**B1**		
2(b)	2880	**B1**		
2(c)	$\frac{x}{8} = 120$	**M1**		• Allow $\frac{x}{8} = -120$
	$x = \pm 960$ and no other solutions in $-1080 \leqslant x \leqslant 1080$	2	**A1** for each, ignoring extras	• One correct answer implies M1

Question	Answer	Marks	Guidance	Notes
2(d)	Correct smooth curve with maximum at (0, 6) and which has x-intercepts **between** 900° and 1080° **and** −900° and −1080° 	2	**B1** for attempt at the correct shape with a maximum at $y=6$ and only one turning point OR for attempt at the correct shape with a maximum on the positive y-axis and only one turning point with x-intercepts **between** 900° and 1080° **and** −900° and −1080°	• Condone some poor curvature/flicking out at ends/feathering • For B2 ‘ends’ must be at approximately $(\pm1080,\ 0 < y < -2)$ • Ignore curve outside of domain $-1080 \leqslant x \leqslant 1080$
3(a)	$a+5d=1.5(a+2d)$ oe soi	B1		• $a = 4d$
	$255=\frac{10}{2}(2a+9d)$ oe soi	B1		• $51 = 2a + 9d$ • If summing individual terms must get as far as simplifying to $255 = 10a + 45d$
	$a = 12$, $d = 3$	2	**M1** for solving *their* simultaneous equations in a and d as far as $a = \ldots$ or $d = \ldots$ providing at least **B1** awarded	• For M1 must have a correct elimination of one unknown for *their* equations and find a value for a or d, even if the subsequent method has slips • If ‘255’ is consistently misread as 225, give B1 B0 MR, B2 for $d=\frac{45}{17}$ oe **and** $a=\frac{180}{17}$ oe accept $d=2.647\ldots$, $a=10.588\ldots$ to 3 or more sf

Question	Answer	Marks	Guidance	Notes
3(b)	$\frac{n}{2}\left(24+3(n-1)\right)*40000$ oe where * is any inequality sign or =	**M1**	For use of *their* values of a and d in a correct sum formula $\frac{n}{2}\left(2\times their\ a+their\ d\times(n-1)\right)*40\,000$	• May be implied by clear comparison of $\frac{n}{2}\left(2\times their\ a+their\ d\times(n-1)\right)$ with 40 000 in e.g. trial and improvement • FT MR of '225': $\frac{n}{2}\left(2\times\frac{180}{17}+\frac{45}{17}(n-1)\right)*40000$
	$3n^2+21n-80000*0$ oe	**A1**		• Writes in solvable form • FT MR of '225' – must be exact fractions: $\frac{45n^2}{34}+\frac{315}{34}x-40000*0$ oe
	Solution of *their* quadratic equation	**M1**	Attempt to solve *their* quadratic in form $ax^2+bx+c=0$ by: factorising – factorisation must result in correct a and c for FT equation, or scaled form of FT equation. quadratic formula – discriminant of FT equation must not be negative. Formula must be correct but condone one sign error in substitution. completing the square – must get as far as $(x+\frac{b}{2})^2=-\frac{c}{a}+\frac{b^2}{4}$ or equivalent, condone one sign error.	• Allow slips in reaching their quadratic • If calculator used and negative solution also stated then it must be correct: –166.836…. rot to 3 or more s.f. e.g. –167 or –166 • FT MR of '225' $170.38, [x=-177.38]$

Question	Answer	Marks	Guidance	Notes
3(b)	$n = 160$ nfww	**A1**	After **M1 A0**, **SC1** for $n = 160$ from trial and improvement provided **BOTH** $n = 159$ **and** 39591 **AND** $n = 160$ **and** 40080 shown	• Award 4 marks for $3n^2 + 21n * 80000$ • followed by correct answer providing there is no incorrect working • n = 159 or 159.8… or n*160 imply M1A0 (where * is any inequality sign) • Condone $n \approx 160$ • FT MR of '225': $n = 171$ A1FT MR is dependent on all previous marks
4(a)	$\ln 3y = mx^2 + c$ soi	**B1**		• Implied by e.g. $\ln 3y = their mx^2 + their c$ or $\ln 3y - 12.24 = their\, m\left(x^2 - 4.64\right)$ or $\ln 3y - 8.94 = their\, m\left(x^2 - 6.84\right)$
	$m = -1.5$ OR Correct method which could be used to find m e.g. $\dfrac{12.24 - 8.94}{4.64 - 6.84}$ oe or $2.2m = -3.3$ oe	**M1**	**FT** *their c* if found first	• From solving 12.24 = 4.64m + c and 8.94 = 6.84m + c
	Correct method which could be used to find c e.g. 12.24 = (*their* –1.5)4.64 + c or 8.94 = (*their* –1.5)6.84 + c	**M1**	**FT** *their m* if found first	• e.g. $\ln 3y - 12.24 = -1.5\left(x^2 - 4.64\right)$ or $\ln 3y - 8.94 = -1.5\left(x^2 - 6.84\right)$ or $Y - 12.24 = -1.5\left(X - 4.64\right)$ (condone use of y, x) • Implies B1

Question	Answer	Marks	Guidance	Notes
4(a)	$m=-1.5,\ c=19.2$	**A1**		• $c=\dfrac{96}{5}$ • Allow embedded e.g. $Y=-1.5X+19.2$
	$3y=\mathrm{e}^{their\,19.2+(their\,-1.5)x^2}$	**M1**	**FT** *their* m and c	• $3y=\mathrm{e}^{19.2-1.5x^2}$
	$y=\dfrac{\mathrm{e}^{19.2}}{3}\mathrm{e}^{-1.5x^2}$ oe cao	**A1**		• e.g. $y=\dfrac{1}{3}\ \mathrm{e}^{\frac{96}{5}}\mathrm{e}^{-\frac{3}{2}x^2}$ • Allow $a=-\dfrac{3}{2}, k=\dfrac{\mathrm{e}^{19.2}}{3}$ • isw changing $\dfrac{1}{3}\ \mathrm{e}^{\frac{96}{5}}$ into 72 666 258.23
4(b)	99.6[22…]	**B1**		• Allow $\dfrac{\mathrm{e}^{5.7}}{3}$
4(c)	Correct method to find x using *their* equation	**M1**	**FT** *their* equation of the form $y=\dfrac{\mathrm{e}^{their\,19.2}}{3}\mathrm{e}^{(their\,-1.5)x^2}$ or $y=\dfrac{1}{3}\mathrm{e}^{their\,19.2+(their\,-1.5)x^2}$	• If FT need to see the method not just decimals e.g. $x=\sqrt{\dfrac{1}{(their\,-1.5)}\ln\left(\dfrac{30}{\mathrm{e}^{their\,19.2}}\right)}$ • Allow solving for x from $\ln 30=their\,-1.5x^2+their\,19.2$
	±3.25 or ± 3.245[38…]	**A1**		• ± 3.2453867 • One correct value rot 3 s.f. implies M1

Question	Answer	Marks	Guidance	Notes
5(a)	$15\tan^2 5x\left(1+\tan^2 5x\right)$ nfww isw	4	**B3** for $3\left(\tan 5x\right)^2\left(5\sec^2 5x\right)$ oe or $k\tan^2 5x\left(1+\tan^2 5x\right)$ or **B2** for $k\left(\tan 5x\right)^2\left(5\sec^2 5x\right)$ oe or $3\left(\tan 5x\right)^2\left(k\sec^2 5x\right)$ oe where k is a positive constant, k may be 1 or **B1** for $\frac{\mathrm{d}}{\mathrm{d}x}\left(\tan 5x\right)^3 = 3\left(\tan 5x\right)^2 \times \mathrm{f}(x)$ or $\frac{\mathrm{d}}{\mathrm{d}x}\left(\tan 5x\right) = 5\sec^2 5x$ soi	• **May be less simplified for any or all marks** • $\mathrm{f}(x)$ could be a constant or 1 • B1 soi by e.g. $\frac{\mathrm{d}}{\mathrm{d}x}\left(\tan 5x\right)^3 = 3\left(5\sec^2 5x\right)^2$ • May use quotient rule for $y=\frac{\sin^3 5x}{\cos^3 5x}$ **B3** for $\frac{15\sin^2 5x}{\cos^4 5x}$ (must be simplified) or **B2** for $\frac{k\sin^2 5x}{\cos^4 5x}$ where k is a positive constant (must be simplified) or **B1** for $\frac{\mathrm{d}}{\mathrm{d}x}\left(\cos 5x\right)^3 = 3\left(\cos 5x\right)^2 \times \mathrm{f}(x)$ or $\frac{\mathrm{d}}{\mathrm{d}x}\left(\sin 5x\right)^3 = 3\left(\sin 5x\right)^2 \times \mathrm{f}(x)$ soi $\mathrm{f}(x)$ could be a constant or 1

Question	Answer	Marks	Guidance	Notes
5(b)	$\tan^2 5x = 0$ soi	B1	**FT** $k\tan^2 5x = 0$ where k is a positive constant providing their $\frac{dy}{dx}$ is of the form $k(\tan 5x)^2(5\sec^2 5x)$ oe or $3(\tan 5x)^2(k\sec^2 5x)$ oe	• No solutions from $\sec^2 5x = 0$ or $1+\tan^2 5x = 0$ for this mark
	Any two of $5x = 0,\ \pi, 2\pi,\ 3\pi$ soi	M1	**FT** $k\tan^2 5x = 0$ where k is a positive constant	• Ignore any solutions from $\sec^2 5x = 0$ or $1+\tan^2 5x = 0$ for this mark
	$x = 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}$ oe isw or $x = 0, 0.628[3\ldots], 1.25[6\ldots], 1.88[4\ldots]$ and no other solutions in $0 \leqslant x < \frac{4\pi}{5}$	A3	**FT** $k\tan^2 5x = 0$ where k is a positive constant **A2FT** for two correct solutions, ignoring extras; **FT** $k\tan^2 5x = 0$ where k is a positive constant or **A1 FT** for one correct solution, ignoring extras; **FT** $k\tan^2 5x = 0$ where k is a positive constant	• Allow greater accuracy for decimals • For A2 or A1: Ignore any solutions from $\sec^2 5x = 0$ or $1+\tan^2 5x = 0$ • For A3: There must be no solutions from $\sec^2 5x = 0$ or $1+\tan^2 5x = 0$ • Ignore extras outside $0 \leqslant x < \frac{4\pi}{5}$ whether correct or not • After B1, two correct solutions imply M1

Question	Answer	Marks	Guidance	Notes
6	3696	3	**M2** for correct method to find all possibilities e.g. $5 \times 8 \times 7 \times 6 \times 1 + 6 \times 8 \times 7 \times 6 \times 1$ oe $5 \times 8 \times 7 \times 6 \times 2 + 1 \times 8 \times 7 \times 6 \times 1$ oe OR **M1** for $5 \times 8 \times 7 \times 6 \times 1$ oe or $6 \times 8 \times 7 \times 6 \times 1$ oe or $5 \times 8 \times 7 \times 6 \times 2$ oe or $1 \times 8 \times 7 \times 6 \times 1$ oe	• Allow other complete valid methods for M2 e.g. $5 \times {}^8P_3 \times 1 + 6 \times {}^8P_3 \times 1$ or $5 \times {}^8P_3 \times 2 + {}^8P_3$ oe or $11 \times {}^8P_3$ or $10 \times {}^8P_3 + {}^8P_3$ or $11 \times 8 \times 7 \times 6 \times 1$ or $1680 + 2016$ or $3360 + 336$ • M1 for 1680 or 2016 or 3360 or 336 • or $5 \times {}^8P_3$ or $6 \times {}^8P_3$ or $10 \times {}^8P_3$ or $1 \times {}^8P_3$ • For M1 allow embedded as part of a sum or difference but not a product
7	$64 - 576x + 2160x^2$ oe, soi and $1 + \frac{1}{2x} + \frac{1}{16x^2}$ soi	B3	**B2** for $64 - 576x + 2160x^2$ oe, soi OR **B1** for two correct terms in $64 - 576x + 2160x^2$ oe, soi **B1** for $1 + \frac{1}{2x} + \frac{1}{16x^2}$	• Ignore extra terms • Allow less simplified e.g. $2^6 - 18 \times 2^5 x + 9 \times 15 \times 2^4 x^2$ • or as part of full expansion • Allow as separate terms in a list e.g. $64, -576[x], 2160[x^2]$ or $2^6, 18 \times 2^5 [x], 9 \times 15 \times 2^4 [x^2]$ • Accept e.g. ${}^6C_0 \times 2^6 \times (-3x)^0 + {}^6C_1 \times 2^5 \times (-3x) + {}^6C_2 \times 2^4 \times (-3x)^2$ • $1 + \frac{1}{2x} + \left(\frac{1}{4x}\right)^2$ is not sufficient for the B1 but accept $1^2 + \frac{2}{4x} + \frac{1}{16x^2}$

Question	Answer	Marks	Guidance	Notes
7	$\textit{their } 64 - \frac{\textit{their } 576}{2} + \frac{\textit{their } 2160}{16}$	**M2**	**FT** *their* expansions for **M2** or **M1** **M1 FT** for any two correct terms in a sum	• If correct 64 – 288 + 135 OR $2^6 - 18 \times 2^5 \times \frac{2}{4} + 9 \times 15 \times 2^4 \times \frac{1}{16}$ OR $64 - \frac{576}{2} + \frac{2160}{16}$ OR $2^6 + \left(\frac{-576}{2}\right) + \frac{2160}{16}$ • May be unsimplified e.g. $2^6 + ({}^6C_1 \times 2^5 \times (-3) \times \frac{2}{4}) +$ $({}^6C_2 \times 2^4 \times 9 \times \frac{1}{16}$ • If FT values of correct magnitude with incorrect signs, then method marks can be awarded e.g. M2 can be given for 64 + 288 + 135 without calculations seen. If FT any values of incorrect magnitude, calculations **must** be seen. • For M1 FT accept e.g. *their*64 – *their*288 + 1 • If correct, implies previous B marks
	– 89	**A1**		• A correct value is likely to imply full marks unless it is **clearly** from wrong working; look for – 89 and e.g. 64 – 288 + 135 first. • Allow recovery from slips/miscopies if any are seen and solution is otherwise sound

Question	Answer	Marks	Guidance	Notes
8(a)	$AP = \sqrt{20x + x^2}$ oe soi or $AP = 18 - x$ oe soi	**B1**		• Check next to diagram • Allow e.g. $AP = \sqrt{(10+x)^2 - 10^2}$ • Allow e.g. y for AP or CP • If using perimeter '18' must be seen
	$10 + 10 + x + \sqrt{their\left(20x + x^2\right)} = 38$ or $10^2 + \left(their18 - x\right)^2 = \left(10 + x\right)^2$	**M1**	*their* AP must come from valid method using either Pythagoras' or perimeter	• If their AP from Pythagoras it may not be fully simplified e.g. $\sqrt{(10+x)^2 - 10^2}$ • Longer correct methods may be seen For reference: CP = 15.7857[1…] and $AP = \frac{171}{14} = 12.214\ldots$
	Correct solution of *their* equation in x	**M1**	**dep** previous **M1**; **FT** $10 + 10 + x + \sqrt{their\left(20x + x^2\right)} = 38$ or $10^2 + \left(their18 - x\right)^2 = \left(10 + x\right)^2$	• Condone one slip in expansion of $(10+x)^2$ or $(their18 - x)^2$ (i.e. 2 correct terms) • If FT *their* equation in x must be in correct form • If correct: $324 - 36x = 20x \rightarrow x = \frac{324}{56}$
	x = 5.7857[1…]	**A1**		• Accept $x = \frac{81}{14}$ oe or 5.7857 rot 3 s.f.

Question	Answer	Marks	Guidance	Notes
8(a)	$\cos\theta = \dfrac{10}{10 + \mathit{their} x}$ oe	M1	**dep** previous **M1**	• **Must** be seen as this is a 'Show that' question • Accept $\sin\theta = \dfrac{\sqrt{(\mathit{their} x)^2 + 20(\mathit{their} x)}}{10 + \mathit{their} x}$ or $\tan\theta = \dfrac{\sqrt{(\mathit{their} x)^2 + 20(\mathit{their} x)}}{10}$ or $\cos\theta = \dfrac{10^2 + (10 + \mathit{their}\ x)^2 - \mathit{their}\ AP^2}{2 \times 10 \times (10 + \mathit{their}\ x)}$ • If FT substitution of *their x* must be seen • If correct $\cos\theta = \dfrac{140}{221}$, $\sin\theta = \dfrac{171}{221}$, $\tan\theta = \dfrac{171}{140}$
	$\theta = 0.8847... = 0.885$ (to 3 dp)	A1		• **Must see** 4 dp or better first
8(b)	8.85 or 8.847[48…]	B1		
8(c)	44.2 or 44.3 or 44.23[74…]	B1		• Accept 44.25

Question	Answer	Marks	Guidance	Notes
9	$\sin(\theta - 0.5) = 0.4$	**B1**		
	Any one of $\theta - 0.5 =$ 0.411[51…], 2.73[00…], –3.55[31…] soi	**M1**		• Accept values stated to 2 sf • Candidates working in degrees do not earn any method marks until they convert their values to radians
	[θ=] 0.912 or 0.9115[1…] 3.23 or 3.230[0…] –3.05 or –3.053[1…] and no other solutions in $-3.5 \leqslant \theta \leqslant 3.5$	**A3**	**A2** for two correct solutions, ignoring extras or **A1** for one correct solution, ignoring extras	• Ignore extras outside $-3.5 \leqslant \theta \leqslant 3.5$ whether correct or not • 0.9115168… 3.230075… –3.053109… • One correct value implies M1 • Mark at most accurate and ignore subsequent rounding

Question	Answer	Marks	Guidance	Notes
10	$\frac{\mathrm{d}y}{\mathrm{d}x} = 2x\ln\left(3x^2+5\right) + x^2\left(\frac{1}{3x^2+5} \times 6x\right)$ oe, isw	3	**B1** for $\frac{1}{3x^2+5} \times 6x$ **M1** for correct structure of product rule **A1 FT** correct use of product rule **FT** *their* $\frac{1}{3x^2+5} \times 6x$, isw	• For M1 must be a sum with u and v correct but du and dv may both be incorrect
	$x = -1 \rightarrow y = \ln 8$ soi	**B1**		• Accept 2.07944… rot to 3 or more sf
	Substitutes $x = -1$ into *their* $\frac{\mathrm{d}y}{\mathrm{d}x}$	**M1**	**dep** on an attempt at a derivative using product rule	• **If correct,** $\frac{\mathrm{d}y}{\mathrm{d}x} = -2\ln 8 - \frac{6}{8}$ or –4.908883… rot to 2 or more sf implies M1 • Condone slips in substitution, mark intent • **M0 if correct value stated but no evidence of correct derivative**
	Gradient of normal: $\frac{-1}{their\,\frac{dy}{dx}\Big\vert_{x=-1}}$	**M1**	**FT**	• If correct, $\frac{-1}{\left(-2\ln 8 - 0.75\right)}$ or $\frac{4}{3+8\ln 8}$ • accept 0.203712… rot to 2 or more sf • If FT a decimal must see $\frac{-1}{their\text{gradient of tangent}}$

Question	Answer	Marks	Guidance	Notes
10	$y - their\ln 8 = \dfrac{-1}{their\left(-2\ln 8 - 0.75\right)}(x+1)$	M1	**FT** *their* y (*their* $y \neq 0$) and $\dfrac{-1}{their\dfrac{dy}{dx}\Big\vert_{x=-1}}$	• Any correct form e.g. if correct, $y = \dfrac{4x}{3+8\ln 8} + \ln 8 + \dfrac{4}{3+8\ln 8}$, $y - 2.079 = 0.2037(x + 1)$, $y = 0.204x + 2.28$, $\dfrac{y-2.079}{x+1} = 0.2037$, $\dfrac{y-\ln 8}{x+1} = \dfrac{1}{2\ln 8 + 0.75}$ • If using $y = mx + c$, award M1 for $y = \dfrac{-1}{their\left(-4.9\ldots\right)}x + c$ **<u>and</u>** $their\ln 8 = \dfrac{-1}{their\left(-4.9\ldots\right)}(-1) + c$
	Uses $y = 0$ in *their* normal equation and rearranges to $x = \ldots$ e.g. $x = -11.21$ and uses $x = 0$ in *their* normal equation and rearranges to $y = \ldots$ e.g. $y = 2.283$	M2	**dep** previous **M1** **M1 dep** for each	• Ignore any labelling as P and Q even if incorrect • If correct, accept x = awrt –11.2 and y = awrt 2.28 • If FT *their* normal in the form $y = mx + c$ accept y = *their* c. Otherwise substitution and rearrangement must be seen. • Condone $x = 11.2$
	Area of triangle: 12.8 cao	A1		

Question	Answer	Marks	Guidance	Notes
11	At C, $x=\sqrt{3}-1$	**B1**		• Check diagram
	$x^2+x-12=0$	**B1**		• Equates and rearranges to solvable form e.g. $12\left(\frac{1}{1+x}\right)^2+\frac{1}{1+x}-1=0$ or $(1+x)^2-(1+x)-12=0$ May use substitution e.g. $u=1+x \Rightarrow u^2-u-12=0$ • May find a cubic $x^3+2x^2-11x-12=0$
	Solves or factorises *their* 3-term quadratic in x	**M1**	Attempt to solve *their* quadratic in form $ax^2+bx+c=0$ by: factorising – factorisation must result in correct a and c for FT equation, or scaled form of FT equation. quadratic formula – discriminant of FT equation must not be negative. Formula must be correct but condone one sign error in substitution. completing the square – must get as far as $(x+\frac{b}{2})^2=-\frac{c}{a}+\frac{b^2}{4}$ or equivalent, condone one sign error.	• Or factorises *their* cubic equation in x If cubic correct $x=3[x=-4, x=-1]$ For M1 FT, x^3+ax^2+bx+c $=(x+1)(x+d)(x+e)$ where $\lvert de\rvert=\lvert c\rvert$
	At B, $x=3$	**A1**		• If correct $x=3$ $[x=-4]$

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11	Correct plan soi e.g. $\int_0^{their3}\left(3+\frac{1}{1+x}\right)\mathrm{d}x \quad -\int_{their\left(\sqrt{3}-1\right)}^{their3}\left(4-\frac{12}{(1+x)^2}\right)\mathrm{d}x$ or $\int_0^{\left(\sqrt{3}-1\right)}\left(3+\frac{1}{1+x}\right)\mathrm{d}x$ $+\int_{their\left(\sqrt{3}-1\right)}^{their3}\left(\left(3+\frac{1}{1+x}\right)-\left(4-\frac{12}{(1+x)^2}\right)\right)\mathrm{d}x$	**M1**		• Must see their numerical limits for a plan
	$\int\left(3+\frac{1}{1+x}\right)\mathrm{d}x:\ 3x+\ln(1+x)$ **and** **either** $\int\left(4-\frac{12}{(1+x)^2}\right)\mathrm{d}x:\quad 4x+\frac{12}{1+x}$ **or** $\int\left(-1+\frac{1}{1+x}+\frac{12}{(1+x)^2}\right)\mathrm{d}x:$ $-x+\ln(1+x)-\frac{12}{1+x}$	**B3**	**B1** for sight of $\int\left(\frac{1}{1+x}\right)\mathrm{d}x:\ \ln(1+x)$ **B1** for sight of $\int\left(\frac{12}{(1+x)^2}\right)\mathrm{d}x:$ $-\frac{12}{1+x}$	• Condone omission of brackets if recovered

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11	For correct use of limits at least once e.g. $\left[3x+\ln(1+x)\right]_0^{their 3}=9+\ \ln 4-(0+\ \ln 1)$ **or** $\left[4x+\dfrac{12}{1+x}\right]_{their(\sqrt{3}-1)}^{their 3}=$ $\left(12+\dfrac{12}{4}\right)-\left(4\sqrt{3}-4+\dfrac{12}{\sqrt{3}}\right)$	**M1**	**dep** previous **M1**	• Accept e.g. $3\times3+\ln(3+1)$ • M0 if only decimals seen, but isw decimals after exact values seen • 9 + ln4 • $19-8\sqrt{3}$
	Exact area shaded region : $\ln 4+8\sqrt{3}-10$ oe nfww	**A1**		• Must be exact • Ignore decimal (for reference: 5.2427…) if also stated • e.g. $\ln 4+\dfrac{24}{\sqrt{3}}-10$